End of Programme Report on

Mathematical Aspects of Quantum Integrable Models in and out of Equilibrium

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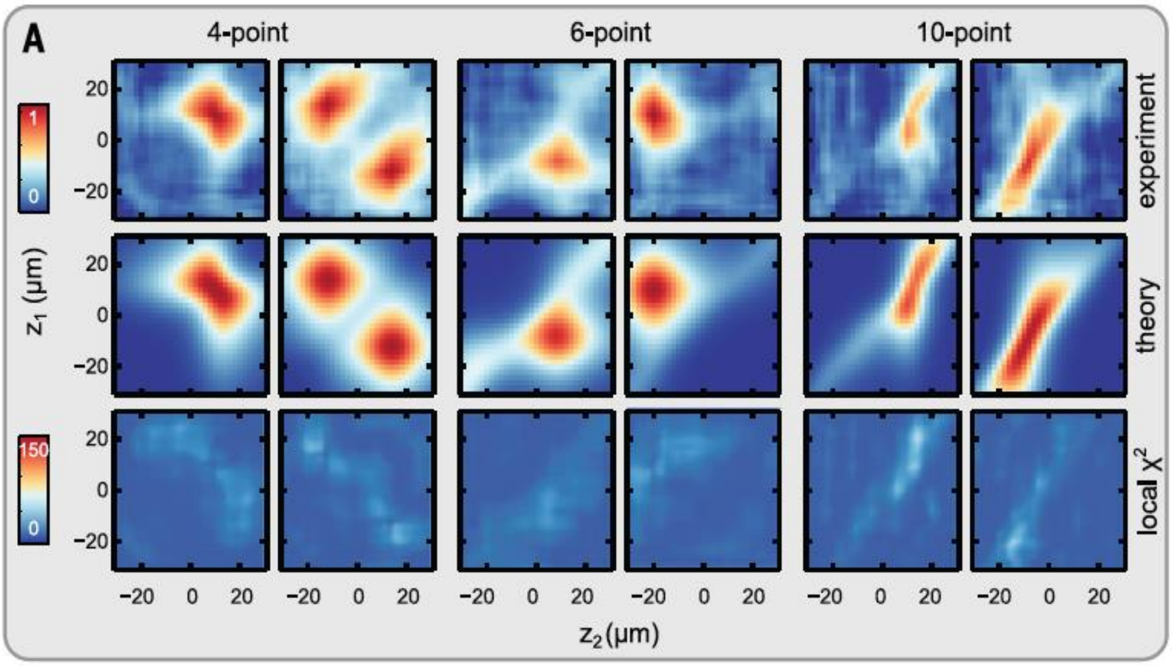

Figure: Multi-point correlation functions out of equilibrium in a one-dimensional cold-atom system (Courtesy of Jörg Schmiedmayer)

Quantum Theory provides the foundation of our understanding of Physics on the (sub) atomic level. While its basic laws are well understood, many key applications face an enormous computational complexity due to the huge number (≈10^{23}) of strongly interacting particles involved. *Quantum integrable models* (QIM) are a particular class of theories, which allow for exact solutions of such problems.

Quantum Integrable Models (QIM) are special, they have long had important applications to e.g. the description of magnetism in solids. Recently they have attracted much interest in relation to experiments on trapped, ultra-cold atomic gases. These are experimentally highly tuneable, and afford the observation of the quantum mechanical time evolution after the atoms have been driven out of their equilibrium configuration. Importantly, the atomic clouds remain to a high degree isolated from their environments for long times. This finally permits the study of fundamental questions, first posed in the early days of Quantum Theory, such as how statistical descriptions of macroscopic systems emerge out of the basic laws of Quantum Theory.

The aim of the programme was to advance QIM methods to address experimentally relevant questions in both equilibrium and non-equilibrium settings. The programme brought together theoretical physicists, mathematicians, and cold atom experimentalists to discuss issues including:

- *Non-equilibrium time evolution in QIM vs generic system.* Constants of motion in QIM constrain their dynamics, and lead to different statistical descriptions at late times. Progress was made on identifying which conservation laws most influence local properties, and how to construct statistical ensembles describing steady state properties. The characterization of these ensembles by existing experimental techniques was discussed, as were methods for accessing the out-of-equilibrium behaviour of weakly perturbed integrable systems.
- *Return amplitudes.* The probability amplitude for a finite quantum system to return to its initial state after a time *t* has interesting properties. In particular, it can exhibit non-analyticities that show a surprising degree of robustness with respect to changing system parameters. Initiatives to better understand the nature of these features were initiated.
- *Mathematical aspects of QIM.* Much of the recent progress in applying QIMs to experimentally relevant questions was based on the *Bethe ansatz*. To generalize these results to related problems, other methods are required. The separations of variables approach was identified as a promising candidate, and avenues for developing it further were identified.

- *Properties of entanglement entropies.* Discussions centred on how entanglement can be used to characterize the spreading of correlations, and on using tensor network methods to study relaxation in generic and integrable systems after quantum quenches.

- *Quantum Transport.* The crossover from ballistic to diffusive behaviour, non-equilibrium steady state properties of externally driven and inhomogeneous, quenched systems, and the shape of flow profiles in out-of-equilibrium systems were discussed in detail. Connections with the important problem of many-body localization were addressed.

The programme led to numerous new collaborations as well as applications for related/follow-up events at the Simon's Centre in Stony Brook and the Erwin Schrödinger Institute in Vienna.