

# NONLINEAR WATER WAVES

(31.VII-25.VIII 2017)

*Report from the organisers: A. Constantin, J. Escher and H. Okamoto*

## 1. SCIENTIFIC BACKGROUND

For more than two centuries progress in the study of water waves proved to be interdependent with innovative and deep developments in theoretical and experimental directions of investigation. In recent years considerable progress was achieved towards the understanding of waves of large amplitude. Within this setting one can not rely on linear theory as nonlinearity becomes an essential feature. Various analytic methods were developed and adapted to come to terms with the challenges encountered in settings where approximations (such as those provided by linear or weakly nonlinear theory) are ineffective. Without relying on simpler models, progress becomes contingent upon the discovery of structural properties, the exploitation of which requires a combination of creative ideas and state-of-the-art technical tools. The successful quest for structure often reveals unexpected patterns and confers to some of these studies aesthetic value.

The purpose of the programme was to bring mathematical analysts and applied mathematicians together, along with engineers, in a venue which will focus on active areas of study of surface water waves of large amplitude where considerable advances were achieved in the last few years. The topics covered in this programme were both multidisciplinary and interdisciplinary, with a strong interplay between mathematical analysis, numerical computation and experimental/field data, interacting with each other via mutual stimulation and feedback.

## 2. STRUCTURE OF THE PROGRAMME

The participation to this four-week programme was global, with representatives attending from Austria, Brazil, Canada, Finland, France, Germany, Ireland, Israel, Japan, Netherlands, Norway, Romania, Russia, Sweden, Switzerland, USA, and the UK. A complete list of short and long stay participants can be found on the INI webpage. A workshop took place in the second week (7th August 2017 to 10th August 2017), featuring talks by A. Abrashkin (Nizhny Novgorod University, Russia), O. Bokhove (Leeds University, UK), A. Chabchoub (Aalto University, Finland), J. Grue (University of Oslo, Norway), B. Harrop-Griffiths (New York University, USA), D. Henry (University College Cork, Ireland), T. Iguchi (Keio University, Japan), A. Ionescu (Princeton University, USA), R. Ivanov (Dublin Institute of Technology, Ireland), R. S. Johnson (Newcastle University, UK), V. Kozlov (Linköping University, Sweden), Y. Matsuno (Yamaguchi University, Japan), A. Nachbin (IMPA, Brazil), H. Okamoto (Gakushuin University, Japan), E. I. Parrau (University of East Anglia, UK), M. Shoji (Japan Women's University), M.

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Stassnie (Technion, Israel), W. Strauss (Brown University, USA), G. Thomas (University College Cork, Ireland), T. van den Bremer (University of Edinburgh, UK), E. Varvaruca (University of Iasi, Romania), V. Vladimirov (University of York, UK), S. Walsh (University of Missouri, USA), M. Yamada (Kyoto University, Japan). Talks by H. Holden (Norwegian Institute of Science and Technology), G. Haller (ETH Zürich, Switzerland), M. Hadzic (King's College London, UK), D. Clamond (University of Nice, France) and E. Wahlén (University of Lund, Sweden) took place in the third and fourth week of the programme. The overall range and depth of these listed lectures was extraordinary, covering analytical aspects, experimental results and numerical simulations; remote access to streamed lectures is available on the INI webpage. These talks were blended in with large time gaps for informal discussion groups. The formal dinner was held on Wednesday, 9th August, at Emmanuel College.

### 3. OUTCOME AND ACHIEVEMENTS

One of the most interesting outcomes of the programme was the interaction between researchers working primarily on physical aspects of water wave theory and those working on theoretical underpinnings. Another disciplinary divide which was bridged is the experimental viewpoint versus the theoretical viewpoint. A good indication of the volume of activity at the programme, and of the areas identified for future research emphasis is the collection of contributions in the special issue of the *Philosophical Transactions of the Royal Society A: Mathematical, Physical & Engineering Sciences*, devoted to the programme (Volume 376, issue 2111, January 2018). This special issue collects various papers authored by some of the participants with the aim to offer a snapshot of the current research activity in the broad field of water waves. The authors consider water waves from different points of view: some papers are concerned with small-amplitude motions, while others tackle waves of large amplitude. A wide range of mathematical methods and modelling approaches (analytical, numerical, as well as experimental) is covered, giving a broad overview of topics of current interest. Recent progress on classical aspects (for example, particle paths) as well as on modern aspects (for example, the modelling of tsunamis) is reported.

The programme did bring successfully different communities together: among the participants there were not only pure and applied mathematicians, but also researchers that have engineering and, in particular, experimental expertise (A. Chabchoub, J. Grue, M. Stiassnie, G. Thomas, T. van den Bremer). Their participation broadened considerably the interdisciplinary character of the programme. The scientific activities during the programme nurtured breakthroughs in the multidisciplinary study of water waves by means of discussions of the state-of-the-art and by setting forth some new promising directions of research. The research results and specific outputs that illustrate this are presented in the special issue of the *Phil. Trans. Roy. Soc. A* mentioned above, some of the highlights being:

- high-precision numerical simulations for travelling waves in flows with constant non-zero vorticity over a flat bed draw attention to the considerable differences that exist between these flow patterns and the case of irrotational water-wave propagation;
- theoretical aspects and experimental studies devoted to the study of the pressure beneath a surface wave;

- the issue of long-time existence of solutions of small amplitude for surface waves in deep water;
- the Stokes drift phenomenon;
- the propagation of irrotational water waves over variable bottom
- the Coriolis effect due to the Earth's rotation on tsunami waves;
- nonlinear studies of geophysical flows (waves and currents);
- computations of solitary hydroelastic waves;
- the statistical description of random surface wave interaction on deep water.

We believe that the programme did benefit the UK mathematical community in a variety of ways, in particular by enhancing its leading position in some aspects of water wave studies and by opening up new directions that were pioneered outside the UK. We expect that some of the discussions that were initiated during the programme will lead to long-term fruitful collaborations resulting in major publications and further scientific activities and advances.