

Mathematical Challenges in Quantum Information; 27 August – 20 December 2013

The crucial role of information processing and communication in modern society makes the transformative potential of Quantum Information science for 21st century technology immense. The subject of Quantum Information ranges from the foundations of quantum mechanics and the physical limitations of information processing to technological issues such as the exploitation of quantum physics, to obtain exponentially enhanced computing power and novel possibilities for communication and information security. It is a highly cross-disciplinary field with essential inputs from computer science, information theory, quantum physics, engineering and mathematics, include the use of random states and operations, methods from operator theory, functional analysis and convex geometry. Although the range of techniques already used is wide, expertise is dispersed and it is clear that other areas of mathematics have much to contribute.

Indeed the need to improve the subject's mathematical connectivity was a key motivation for the programme which was built around the participation of international experts in quantum information theory. With 115 visiting fellows, it is impossible to record all the discussions that took place and their impact. However the hoped for increase in mathematical sophistication was quickly realised. Indeed many of the key breakthroughs were made by younger researchers, new to the field who brought new perspectives from different backgrounds. To illustrate the general picture here are two examples: one concerns an important scientific breakthrough and the other recalls a chance encounter between different communities who interacted at the Institute.

A scientific highlight of the programme was the resolution of the “Gaussian optimizer conjecture”, whose proof by visiting fellows Raul Garcia-Patron, Vittorio Giovannetti and Alexander Holevo concluded a more than 40-year quest for the capacity of bosonic quantum channels. The breakthrough followed their unsuccessful attempt during the programme to find a counterexample(!). The new technique which emerged is elegant and yields a refined understanding of bosonic channel capacity beyond its numerical determination. However it does not settle a stronger conjecture, the so-called *Entropy Photon Number Inequality*, thus fuelling renewed interest in this deep question which has broad ramifications in quantum information theory. A Newton Institute Case Study is in preparation.

An unplanned but serendipitous coincidence was that in September/October there was a parallel programme in the Institute on the Mathematics and Physics of the Holographic Principle. This led to a series of very fruitful joint discussions on the black hole firewall question – whether an observer, upon falling into a black hole old enough to be maximally entangled with its environment, would be burned up immediately at the hole's event horizon, rather than surviving to be destroyed later, as previously thought.

There were three workshops. The first made a conscious effort to include speakers from outside quantum information; it did an excellent job in setting the tone of the whole programme.

The second was on quantum marginals, an area where long-standing difficult problems are yielding to new techniques from convex analysis and random matrix theory and there are emerging links to algebraic and symplectic geometry, and asymptotic representation theory. The final meeting was supported by the Heilbronn Institute and it was focussed on issues of quantum computational complexity and algorithms.

A remote lecture was delivered from Stanford University in the USA by Patrick Hayden on “The computational complexity of entanglement detection” and the programme held a “Q+” google hangout, delivered by Renato Renner from ETH Zurich on the topic “Does freedom of

choice imply that the wave function is real?" In addition, a number of programme participants were invited speakers to the "UCL-Paris Quantum Connection", a 2-day meeting at University College of London in November.

Follow up meetings are planned: for example, Andreas Winter (Barcelona), Nilanjana Datta (Cambridge), Renato Renner (ETH Zurich) and Mark Wilde (LSU Baton Rouge) made a successful bid for a week-long workshop "Beyond IID in Information Theory", to be held in July 2015 at the Banff International Research Station (BIRS).

This field is now the subject of major investment by the UK government, news of which was announced during the programme and is now embodied in an EPSRC call for a "National Network of Technology Hubs". The Institute Director and programme participants took part in a consultation exercise with GCHQ to advise on a forthcoming funding call.

