

On a Relevant Aspect in Difference Equations

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Abstract: In this short note, we obtain the solution of the most general linear system of difference. First this was solved in [1], however here we solve it in a much more easy way.

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Consider the infinite set of nonlinear equation

$$a_{j+1} = \sum_{k=0}^j \alpha_k^{j+1} a_k + \sum_{i=1}^{j-1} \sum_{k=1}^{j-1} \gamma_{i,k}^{j+1} a_i a_k + b_{j+1} \quad \text{for } j > 0$$

Introducing

$${}^1a_j = {}^2a_j + {}^2Q_j$$

it turns out ${}^1a_j = {}^2q_j$ for $0 \leq i \leq 2$ because ${}^1a_{j+1}$ only has nonlinear terms for $j \geq 2$.

If we develop a_j as a function of the initial condition a_0 we have

$$\begin{aligned} {}^2Q_{j+1} &= \left(\sum_{r=1}^{j+1} \sum_{(l_1, l_2, \dots, l_r)}^{j+1} \nabla_{(l_1, \dots, l_r)}^{j+1} \right) q_0 \\ &+ \sum_{i=1}^j \left(\sum_{r=1}^{j+1-2} \sum_{(l_1, \dots, l_r)}^{j+1-2} {}^1\nabla_{(l_1, \dots, l_r)}^{j+1} \right) {}^1b_i \} + {}^1b_{j+1} \end{aligned}$$

Where E_r^1 is the set of all r – components vectors (l_1, \dots, l_r) such that

$$\sum_{i=1}^r l_i = l \quad \text{whith} \quad l_i = \text{integer}$$

and

$$\nabla_{(l_1, l_2, \dots, l_r)}^i \cong {}^i\alpha_{j-e_1}^j {}^i\alpha_{j-l_1-l_2}^{j-l_1} {}^i\alpha_{j-l_1-l_2-l_2}^{j-l_1-l_2} \sim {}^i\alpha_{j-l}^{j-\sum_{i=1}^{r-1} l_i}$$

Introduction

The subject of difference equations is a very important topic from the theoretical point of view and mainly for applications. One good reference is the book by Poole [2].

We remember the reader that if we wish to solve the most general linear problem in difference equations, then we have to solve the following one, namely:

$$x_m = a_{m-1}^m x_{m-1} + a_{m-2}^m x_{m-2} + \dots + a_1^m x_1 + a_e^m x_e + b_m^m$$

This has been solved by induction principle by the authors many years ago. However this approach was difficult.

We give a view of the older method. Now here we are solving it in a easy way.

By the way, we would like to say that many specialists in the subject and some paragraphs in advance and common books, say that this general problem is unsolvable. Moreover, many of these specialists criticize it, since in the computer the memory blows up for some complicated problems.

Our old formula was applied successfully in several subjects in particular to mathematical models in biotechnology.

Now, we have found a new form which is easier than the old one.

In the equation (1) if we revert the order, that is to say

$$x_m = a_0^n x_0 + a_1^m x_1 \dots \dots + a_{m-2}^m x_{m-2} + a_{m-1}^m x_{m-1} + b^m$$

and now take the first n terms from $n = 0$ in the form

$$x_1 = a_0^1 x_0 \qquad \qquad \qquad + b^1$$

$$x_2 = a_0^2 x_0 + a_1^2 x_1 \qquad \qquad \qquad + b^2$$

$$x_3 = a_0^3 x_0 + a_1^3 x_1 + a_2^3 x_2 \qquad \qquad \qquad + b^3$$

$$x_{m-2} = a_0^{m-2} x_0 + a_1^{m-2} x_1 + a_2^{m-2} x_2 + \dots \qquad \qquad \qquad + b^{m-2}$$

$$x_{m-1} = a_0^{m-1} x_0 + a_1^{m-1} x_1 + a_2^{m-1} x_2 + \dots + a_{m-2}^{m-1} x_{m-2} \qquad \qquad \qquad + b^{m-1}$$

$$x_m = a_0^m x_0 + a_1^m x_1 + a_2^m x_2 + \dots + a_{m-1}^m x_{m-1} \qquad \qquad \qquad + b^m$$

Now if you take and define the matrix $m \times m$

Introducing

$$\bar{x}(r, s) = \begin{pmatrix} x_r \\ \vdots \\ x_s \end{pmatrix}$$

$s > r$, of length $s - r$, the one of the system is transformed to

$$(\bar{x}(1, m) - b) = A(\bar{x}(0, m - 1)) + b$$

From here we have

$$(\bar{x}(1, m) - b) = A\bar{x}(0, m - 1)$$

Now A is triangular and if the determinant is different of zero, or

$$\prod_{j=1}^m a_{j-1}^j$$

then it is computed and is well defined the inverse of A . We call A^{-1} its inverse. Then multiply from the left we have

$$A^{-1}(\bar{x}(1, m) - b) = \bar{x}(0, m - 1)$$

or

$$-A^{-1}b = \bar{x}(0, m - 1) - A^{-1}\bar{x}(1, m)$$

$$-A^{-1}b = \begin{pmatrix} x_0 \\ x_{m-1} \end{pmatrix} - A^{-1} \begin{pmatrix} x_1 \\ x_m \end{pmatrix}$$

$$-A^{-1}b = \begin{pmatrix} x_0 - A_{11}^{-1}x_1 \\ x_1 - A_{22}^{-1}x_2 \end{pmatrix}$$

This given x_0 all the other x^1 s may be easily determined. Therefore this is a solution more general but simple.

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