Philosophy or not? The study of cultures and practices of mathematics

Benedikt Löwe*

December 14, 2015

Abstract

The most commonly accepted name of our research field is *Philosophy of Mathematical Practice*, giving philosophy a prioritized role among the many disciplines involved in the field. We explore the interplay between philosophy and other disciplines and its effect on the further development of our field.

1 Introduction

The conference *Cultures of Mathematics and Logic* in Guangzhou brought together philosophers, sociologists, historians, cognitive scientists, and researchers in mathematics education; it was one event among many in the past decade that studied cultures and practices of mathematics. The appendix of (Larvor, 2016) in this volume lists these events.

A look at this list reveals that many of them have titles that include the word "philosophy" or philosophical terms such as "epistemology". The community that meets at these events is closely linked to the *Association for the Philosophy of Mathematical Practice* (APMP) which serves as the institutional backbone of the field. Many of the researchers in our field refer to it as *Philosophy of Mathematical Practice* and self-identify as philosophers. A conference participant who is not a philosopher might ask: why does philosophy play such an important role in the field? In contrast, are those events that do not grant philosophy this priority even events in the same field?

In this paper, we shall explore the tension between the central role of philosophy in our community and the fact that philosophy is only one among many fields interested in mathematical cultures and practices. We start in § 2 by exploring the notions of "research field", "community", and "discipline" and argue that, entirely independently of which name we choose for it, we can talk of "our community", even though its members belong to different disciplines and may have different motivations. In § 3, we turn to the name of our community and propose that the occurrence of the term "philosophy" in "philosophy of mathematical practice" indicates a specific goal of research

*Institute for Logic, Language and Computation, Universiteit van Amsterdam, Postbus 94242, 1090 GE Amsterdam, The Netherlands; Fachbereich Mathematik, Universität Hamburg, Bundesstraße 55, 20146 Hamburg, Germany; Corpus Christi College, University of Cambridge, Cambridge CB2 1 RH, England; Isaac Newton Institute for Mathematical Sciences, 20 Clarkson Road, Cambridge CB3 0EH, England. This paper was finished whilst the author was a Simons Visiting Fellow of the Programme *Mathematical, Foundational and Computational aspects of the Higher Infinite* at the Isaac Newton Institute for Mathematical Sciences. The author would like to thank Brendan Larvor, Thomas Müller, and Bart Van Kerkhove for comments.

rather than a claim that the study of mathematical cultures and practices is intrinsically philosophical in nature and explain that the philosophically motivated research on mathematical practices and cultures has been a driving force for our community. In the following two sections, we then discuss the practical consequences of this contingent fact: in § 4, we discuss a number of important distinctions for empirical approaches to philosophy, in particular one due to Prinz; in § 5, we then discuss how the overall research situation in the broader field of studying mathematical practices and cultures has an effect on which of Prinz's approaches we can choose. We conclude in § 6 with a discussion about the consequences of the analysis of our field given in this paper.

2 Our community

Several factors, both intrinsic and extrinsic to the academic and scientific content, are relevant for determining whether a field of research can be classified as a *research community* or even a *discipline*.[1] Among the intrinsic factors are coherence in the subject matter of research, the motivation of the research, and the methods of doing the research; the extrinsic factors include questions such as whether there are research institutes, departments, research programmes, conference series, journals, undergraduate and postgraduate teaching programmes for the field. Sometimes, the status of a field as a discipline is being questioned on some grounds, e.g., that the field does not have a homogeneous research method or its own publication culture. In many cases, when publicly raised, these doubts come with a strategic agenda, either as an attempt to discredit a field or as a rallying cry from inside the field.

In some relevant debates, both the disciplinary nature of a research field and some of the intrinsic factors for disciplinarity become crucial: for instance, in a paper that became the starting point for a lively debate, Stokhof and van Lambalgen (2011) raised the question whether "modern linguistics [is] [...] an example of a 'failed discipline' " (p. 3). Answering such a question presupposes that modern linguistics is a discipline that shares a common list of goals in comparison to which the achievements of the field can be seen as "failure". Maddirala (2014) discusses the issue raised by Stokhof and van Lambalgen for the more narrow field of formal semantics by doing an interview study with formal semanticists; his study shows that discrepancies between the goals and intentions attributed to a field from the outside, those professed by members of the community in public, and those implicitly observable from work of the members of the field make it difficult to determine the precise meaning of questions like the one by van Lambalgen and Stokhof.

2.1 Subject matter.

Taking philosophical issues seriously, uniformity of the subject matter of the field is difficult to achieve since agreement about what the subject matter is presupposes agreement about major questions about the ontology of the field; e.g., saying that mathematics is the study of properties of mathematical objects is taking a strong philosophical position about the ontology of mathematics. Even in established subfields of philosophy such as philosophy of language and philosophy of mind, it would be difficult to get the practitioners of the field to agree on the ontology of their area's subject

[1] In the following, we shall not define either of the three terms "field of research", "research community", or "discipline" precisely. We shall be using "field of research" as the most generic term covering any collection of researchers or research results; the other two terms, "research community" and "discipline" are more specific, but we remain agnostic about their precise definition, as the discussion in this section exhibits.

matter. The only solution to this problem is to suspend the philosophical commitment to scrutinize fundamental terms and leave the subject matter deliberately philosophically underdetermined by calling it, e.g., "language" or "the mind" without giving a definition.

Disciplinary borderlines do not respect the boundaries of metaphysical schools: mathematicians who believe that mathematical objects exist independently from human activity and those who believe that mathematics is just a formal game are equally classified as "mathematicians" and neither of them would refuse to call the other a mathematician. So, when we talk about "subject matter" in the context of the definition of disciplines, we are talking about terms that are philosophically underdetermined.

In our case, we can and do agree that the subject matter is "mathematical practices and cultures" or "mathematical activity as practised by human beings". As soon as we apply philosophical scrutiny to this underdetermined description, the consensus of what the subject matter of our field is will break down, but our field does not differ in this respect from established disciplines such as mathematics, physics, or others.

2.2 Research methods.

It is primarily the uniformity of the research method or the collection of research methods that determine a discipline. Etymologically, the word discipline suggests the body of knowledge or techniques that are taught to the *discipuli* in a rigourous teaching environment:

> [*disciplina*] désigne [...] ce qui fait d'objet de l enseignement, la MATIÈRE ENSEIGNÉE, μάθησις, et spécialement la matière d'un enseignement régulier, systématique, autrement dit une science, ἐπιστήμη. (Marrou, 1934, p. 6)
>
> La notion d'enseignement devient très lontaine; elle finit par s'effacer et *disciplina* signifie règle imposée [et] BON ORDRE. (Marrou, 1934, p. 11)

By definition, interdisciplinary endeavours do not have this uniformity of research methods. Our field, driven by the realisation that one discipline alone cannot give a multidimensional picture of all facets of mathematical practice, is a prototype of such an interdisciplinary endeavour. As a consequence, we cannot and should not expect that the field is defined by methodological coherence. Assuming the standard usage of the word "discipline", we observe that our field cannot be a discipline.

2.3 Motivation.

Even established disciplines do not have a commonly shared motivation. Taking mathematics as an example, representatives of different subfields of mathematics will strongly disagree on what the motivation for their work is. If you organise a panel discussion on the topic of 'What motivates mathematical research?' with a set theorist, a differential geometer, a mathematical physicist, and an actuary, you can expect heated debate and very little agreement. The dispute may even reach the point where some panelists start to doubt whether mathematics is a single discipline, given their fundamental dissensus. Yet, these emotional reactions do not become a serious argument that mathematics is not a discipline; instead, we have to revise our view that a uniform motivation is important for the disciplinary character of a field.

Similarly, we shall observe that members of our field strongly diverge on what their motivations for their research are, even to the point where some feel that the disagreement is so fundamental that we are not dealing with one community, but several. The discussion of our differing motivations

for being interested in "mathematical practices and cultures" is the main focus of §3, so we defer this discussion to later.

2.4 Extrinsic factors.

Many of the mentioned extrinsic factors (existence of research institutes, departments, research programmes, conference series, journals, undergraduate and postgraduate teaching programmes) are difficult to separate from the institutional set-up of our academic world; therefore, there is a strong correlation between extrinsic factors and whether a field is considered a traditional discipline or not. Since the vast majority of the universities, research institutes and funding agencies is organised along the lines of the established disciplinary boundaries, there is something that we could dub the *disciplinary barrier*: institutions make decisions whether to grant a degree, whether to accept a postgraduate student, whether to hire a postdoc or a new faculty member, whether to give tenure or to promote, whether to fund a research project, whether to award a prize or elect to a position of recognition mostly according to the standards of the relevant established discipline. This means that the participants of any interdisciplinary research endeavour will need to play according the disciplinary rules in order to maintain the chances of a research career. This makes it very difficult and—if the discussed field is interdisciplinary by design—possibly undesirable to establish some of the mentioned extrinsic factors that create community cohesion beyond or between the disciplines.

For instance, typically (but not always) a research journal requires submitted papers to use a particular research methodology. Of course, there are generalist journals (such as *Science* and *Nature*) or genuinely interdisciplinary journals (such as the newly founded journal *Computability*), but there are practical reasons why such examples are few and far between. Papers that straddle methodological boundaries are difficult to judge: If a paper uses methods from fields X and Y, does it have to meet the standards of publication in both X and Y, or do we think that the joining of techniques creates intrinsic added value going beyond the sum of its parts? Do we need referees from both X and Y? If they disagree, how do we aggregate their judgments and how do we deal with conflicting judgment styles between the disciplines?

Similar practical issues apply to other extrinsic factors such as postgraduate programmes (does a thesis merit an interdisciplinary degree in X and Y if it is not sufficient for a degree in either X or Y?) or departments and research institutes (do we want to hire a person who would not be strong enough to be hired in a Department of X or in a Department of Y?), and research fields that are interdisciplinary hit what we called the disciplinary barrier.

On the other hand, our field exhibits most, if not all extrinsic indicators for being a community that are just below the disciplinary barrier: conferences are being organised with proceedings volumes that are published as books or special issues of leading research journals, conference series and research networks, often with funding from standard funding agencies, are being established, and, not least, the members of our field decided to form an association, the *Association for the Philosophy of Mathematical Practice*.[2] Those of us who have been to several of the conferences and workshops of the field realize that people they met at one of the conferences also come to others, allowing discussions to extend beyond the confines of a single conference. As usual in interdisciplinary fields every participant thinks that they are on the fringe of the subject and that everyone else has more sense of belonging than they do; but meeting the same people in different configurations at various

[2]Once more, we refer to the appendix of (Larvor, 2016) for a list of these events and activities.

conferences gives a clear sense of cohesion.

We conclude that we are allowed to talk about a *community*: a field that coheres enough that community members collaborate, publish jointly, regularly meet at conferences and workshops, set up networks, but still remain part of their respective disciplines that requires the members to satisfy the research and publication expectations of their different disciplinary backgrounds. It is this community that we wish to understand better in the remaining sections of this paper.

3 Philosophy

In § 2, we have seen that our field is not a *discipline* in the traditional sense, but exhibits many of the extrinsic indicators for being a *research community*. So far, we have been using "our field" to refer to it, deliberately avoiding to give it a name. In this section, we shall now move to the thorny issue of naming our community.

One of the important components of our argument that our field constitutes a community was the fact that the *Association for the Philosophy of Mathematical Practice* forms an institutional backbone for our field. That in turn could be used as an argument that "Philosophy of Mathematical Practice" is the official and accepted term for the field. Does that imply that our field is a subfield of philosophy? Many if not most of the members of our community are philosophers or are at least philosophically interested. This interesting contingent fact about our community asks for an explanation: why is it that philosophers are so interested in studying mathematical cultures and practice?

For many of the philosophers in our community, the reasons for becoming interested in the empirical study of mathematical practice were negative: the foundational debates in the philosophy of mathematics in the early 20th century had resulted in a foundationalist school of thought dealing with a highly idealised version of what mathematics is, embedded in the formalism of first-order logic (or sometimes, second-order logic). Crucial notions of mathematical epistemology such as the notion of "proof" occurred in their sanitized form of formal derivations in a particular formal framework. The role of mathematical logic for the understanding of what mathematics can do and cannot do, should not be underestimated; but on the other hand, philosophers in our community felt that it was equally obvious that the amount of idealisation involved in the transformation from the complex human activity we call mathematics to the crisp and clean formal world of formal derivations hides many important and interesting features of what it is to do mathematics:

> [From the early 20th century onwards,] the foundations of mathematics became a central research interest. This resulted in a widely accepted notion of *formal derivation* as the explication of mathematical proof. [...] In mathematical practice, proofs are written down in a more condensed, semi-formal style. [...] The traditional view would consider these proofs *enthymematic*, leaving out technical detail for purely pragmatic reasons . [...]
>
> A closer look at mathematical practice leads to two important observations. First, the completion of enthymematic, semi-formal proofs to formal derivations almost never happens and hardly plays any rôle in the justification that mathematicians give for their theorems; second, also the production of semi-formal proofs [...] is only the final step of the mathematical research process. This final step, while important for the documentation of results and crucial for the careers of researchers, is not necessary for

> the acceptance of a proof by the mathematical community. For this, different forms of proof are much more relevant: informal sketches on the blackboard, or scribblings and drawing on napkins [...] Shouldn't these forms of proof replace the unrealistic notion of formal derivation in our epistemology of mathematics? (Buldt et al., 2008, pp. 310–311)

This attitude resulted in a number of publications whose very titles suggest that the traditional philosophical account does not deal with the true content of mathematics, e.g., *What is mathematics, really?* (Hersh, 1997) or *Towards a philosophy of real mathematics* (Corfield, 2003). The term "mathematical practices" was used by the Brussels philosophers of mathematics Jean Paul Van Bendegem and Bart Van Kerkhove when they started their series of meetings in 2002,[3] and the term "philosophy of mathematical practice" was chosen by Paolo Mancosu (2008b) as the title of a book containing papers by some of the protagonists of our community. The publication of (Mancosu, 2008b) has to be seen in the context of the formation of the *Association for the Philosophy of Mathematical Practice*, founded in the year 2009 with Mancosu as one of the nine founding members (three of the nine founders contributed to Mancosu, 2008b; three more are cited in Mancosu, 2008a). The Brussels conferences *Perspectives on Mathematical Practices* (October 2002 and March 2007) and their proceedings volumes (Van Kerkhove and Van Bendegem, 2007; Van Kerkhove, 2008; Van Kerkhove et al., 2010) were important contributing factors in the pre-history of the *Association for the Philosophy of Mathematical Practice*.

In his introduction, Mancosu (2008a) contrasts the philosophy represented in his volume with traditional philosophy of mathematics, giving an implicit definition of "philosophy of mathematical practice" in terms of this contrast:

> The contributions presented in this book are [...] joined by the shared belief that attention to mathematical practice is a necessary condition for a renewal of the philosophy of mathematics. (Mancosu, 2008a, p. 2)

He then discusses the "foundational tradition" and the "maverick tradition" representing two movements within philosophy of mathematical practice. The term "maverick tradition" goes back to Aspray and Kitcher[4] and Mancosu characterizes it by

> a. anti-foundationalism, i.e., there is no certain foundation for mathematics; mathematics is a fallible activity; b. anti-logicism, i.e., mathematical logic cannot provide the tools for an adequate analysis of mathematics and its development; c. attention to mathematical practice: only detailed analysis of large and significant parts of mathematical practice can provide a philosophy of mathematics worth its name. (Mancosu, 2008a, p. 5)

[3] However, their acronym PMP stands for "Perspectives on Mathematical Practices" rather than "Philosophy of Mathematical Practice".

[4] Kitcher and Aspray (1988, p. 17) describe the maverick tradition in terms very similar to the other characterisations of our community in this section: "[I]t is pertinent to ask whether there are [...] tasks for the philosophy of mathematics [...] that arise either from the current practice of mathematics or from the history of the subject. A small number of philosophers [...] believe that the answer is yes. Despite large disagreements among the members of this group, proponents of the minority tradition share the view that philosophy of mathematics ought to concern itself with the kinds of issues that occupy those who study other brnaches of human knowledge [...]: How does mathematical knowledge grow? What is mathematical progress? What makes some mathematical ideas [...] better than others? What is mathematical explanation?"

One of the mentioned proponents of the maverick tradition, Jean Paul Van Bendegem (2014), also one of the founders of the Association, gives another list of eight approaches of the philosophy of mathematical practice:

> (a) the Lakatosian approach, also called the 'maverick' tradition; (b) the descriptive analytical naturalizing approach; (c) the normative analytical naturalizing approach; (d) the sociology of mathematics approach; (e) the mathematics educationalists approach; (f) the ethnomathematical approach; (g) the evolutionary biology of mathematics; and (h) the cognitive psychology of mathematics. (Van Bendegem, 2014, p. 221)

A third founder and the current president of the Association, José Ferreirós, drew a picture of the simultaneous diversity and unity of the field during a talk given on 11 September 2014 in Pont-á-Mousson:

> [Philosophy of Mathematical Practice] has different branches [that share] some very basic tenets [...] (the need to complement philosophical analyses with new features emerging from attention to concrete cases, the role for history and present-day studies, the emphasis on methodological differences between areas of math[ematics], the openness to interdisciplinary considerations, etc.) but then there are quite different ways of articulating [these tenets]. The main idea [is] [...] that philosophers have moved away from a static, monolithic, too idealised and simplified version of what mathematics is, towards what may be called 'real math[ematics]' in the sense of different aspects of the work and activities of mathematicians. In this [move] [...], some people have remained more cautious, while others have been more radical in taking into consideration the actual practice of doing math[ematics] by agents [...] or the social network of mathematicians. (Ferreirós, 2014, § 3)

In the first chapter of his book *Mathematical Knowledge and the Interplay of Practices*, Ferreirós (2015) provides a manifesto of what could be called "epistemology of mathematical practice" and describes it as an interdisciplinary endeavour using tools from a plethora of disciplines and applying them to traditional questions from philosophy:

> During the twentieth century, we have seen several different broad currents in this field, which [...] can be reduced to three main types: *foundational* approaches [...], *analytic* approaches [...], and the so-called "*maverick*" approaches [...], which have typically been anti-foundational and focused on history, methodology, and patterns of change. [...] It seems to be the case that a new generation of philosophers of mathematics has arisen whose work is superseding those distinctions. [...] These philosophers engage in an analysis of mathematical practices that incorporates key concerns of the "mavericks", without adopting their anti-foundational, anti-logical orientation. [...] Notice that the new orientation in the philosophy of mathematics is highly interdisciplinary. Some authors emphasize knowledge of mathematics itself and logic [...]; some others stress the role of cognitive science [...] or sociological approaches [...]; and the list goes on, with mathematics education, anthropology, biology, etcetera. (Ferreirós, 2015, pp. 1–2)

All four quotes emphasize that philosophy of mathematical practice is not dealing with entirely new philosophical questions, but is a particular approach to philosophy of mathematics. In this context, it is interesting to note at the inaugural conference of the *Association for the Philosophy*

of Mathematical Practice in Brussels there was a critical discussion of the term "philosophy of mathematical practice".[5] Its syntactic form "philosophy of X" suggests that there is an object "mathematical practice" whose philosophy it is studying. In particular, the name suggests that the field is distinct from "philosophy of mathematics": whereas the latter studies mathematics with philosophical means, the name "philosophy of mathematical practice" could suggest that the former studies only the practice of mathematics, rather than mathematics itself.[6] This view was in general rejected by the participants of the inaugural conference; instead, the consensus was that philosophy of mathematical practice is an approach (or a collection of approaches) to philosophy of mathematics and this view is reflected in the definition of the purview of the *Association for the Philosophy of Mathematical Practice* on its webpage:

> Over the last few years approaches to the philosophy of mathematics that focus on mathematical practice have been thriving. Such approaches include the study of a wide variety of issues concerned with the way mathematics is done, evaluated, and applied, and in addition, or in connection therewith, with historical episodes or traditions, applications, educational problems, cognitive questions, etc. We suggest using the label "philosophy of mathematical practice" as a general term for this gamut of approaches, open to interdisciplinary work.

But are all of the items listed as part of this "gamut of approaches" really approaches to philosophy of mathematics? In Van Bendegem's list of approaches for philosophy of mathematical practice, five of the eight items are

> not, strictly speaking, approaches 'in the philosophy of mathematical practice' [...] They are, rather, five *non*-philosophical perspectives on mathematical practice that are *used* by philosophers of mathematical practice or, more prudently, on which *some* philosophers of mathematical practice *can find relevant* to rely. (Jullien and Soler, 2014, p. 228; emphasis in the original)

In § 2.1, we claimed that the subject matter of our field is "mathematical practices and cultures"; we now observed that there is consensus that philosophy of mathematical practice is not to be considered the philosophy of a separate subject matter, but rather as a particular approach in philosophy of mathematics. How do we reconcile these two contradictory observations?

The discipline of mathematics, seen as a human activity with its cultural particularities and achievements as well as various practices, is a "subject matter" in the philosophically underdetermined sense of § 2.1 and well worthy of study; there are researchers from many disciplines studying this subject matter from various angles. For lack of a better term, one could call this research field *the study of mathematical cultures and practices*. Different researchers in this field of research have different motivations for studying mathematical cultures and practices: a researcher in mathematics education might be motivated by educational questions or even questions about educational policy; a cognitive scientists would be curious about understanding the cognitive processes in doing mathematics in contrast to other cognitive processes; an anthropologist could be fascinated by the

[5] The following argument was mentioned during a round table discussion during the inaugural conference on 10 December 2010.

[6] Other terms than "philosophy of mathematical practice" have been used that avoid this misinterpretation, among them "empirical philosophy of mathematics" (Löwe et al., 2010), "practice-based philosophy of mathematics" (Dutilh Novaes, 2012), "(socio-)empirically informed philosophy of mathematics" (Müller-Hill, 2009, 2011), or "philosophy of real mathematics" (Corfield, 2003).

difference between mathematical cultural practices and those of other, closely related, yet different disciplines; and, of course, the philosopher would be driven by traditional philosophical questions about epistemology and ontology of mathematics. Every researcher in the field must decide on the basis of the motivating questions which occurring phenomena are sufficiently relevant for their work.[7]

The two mentioned seemingly contradictory observations can be reconciled by understanding that they are two descriptions of extensionally very similar research communities: from the point of view of the broader interdisciplinary community that we dubbed *the study of mathematical cultures and practices*, our community is the sub-community motivated by philosophical questions; from the point of view of the larger philosophy of mathematics community internal to philosophy, our community is the sub-community that embraces a particular approach, basing philosophical claims on actual mathematical practice rather than idealisations. As discussed in § 2.3, the fact that the diverging motivations sometimes create feelings of division should not disturb us too much, as this happens in established disciplines as well.[8]

4 Three relevant distinctions for empirical philosophy

Jesse Prinz (2008) introduces a distinction between two approaches of doing philosophy on the basis of empirical data. He calls these approaches *Experimental Philosophy* and *Empirical Philosophy*:

> Some philosophers make use of empirical results that have been acquired by professional scientists. ... These results are used to support or refute philosophical theories. We shall call this approach 'empirical philosophy'. Other philosophers also conduct their own psychological experiments, an approach known as 'experimental philosophy'. (Prinz, 2008, p. 196)

His distinction is largely based on whether the empirical work is done by the philosophers themselves or rather by other scientists ("mining the data" vs. "collecting the data"):

> Empirical philosophy works by citation. Philosophers cite relevant empirical research and use it to argue for philosophical conclusions. (Prinz, 2008, p. 200)

Prinz acknowledges that the distinction is a contingent sociological fact about philosophy,[9] but argues that the two types of philosophers correspond to a natural division of types of philosophical questions:

> I am not trying to suggest that experimental and empirical philosophy *must* differ along the lines I suggest; only that they often do, and that there are reasons for these differences. (Prinz, 2008, p. 197)

[7] Cf. (Löwe, 2014) for an example of a meta-argument for the philosophical relevance of a particular aspect of the practice of mathematics (viz. the use or the rejection of computer technology).

[8] It is interesting to note the programmatic statements cited in this section implicitly or explicitly acknowledge these feelings of division: e.g., Kitcher and Aspray (1988) mention "large disagreements among the members of this group" and Ferreirós (2014) emphasizes that "there are quite different ways of articulating [these tenets]".

[9] Cf. also (Prinz, 2008, p. 200): "This distinction between experimental and empirical philosophy is very rough. ... [T]here are counter-examples in the literature, and the distinction is likely to blur even more in the years to come."

We believe that the choice of terms *empirical philosophy* and *experimental philosophy* is infelicitous. There are at least three relevant distinctions with respect to the use of empirical data that should be considered here: there is a fundamental methodological question of whether philosophy should take empirical data about human activities and cognitive states into account; assuming that empirical data are taken into account, then there is a second methodological question of which empirical techniques are being used to collect this empirical data; and finally, there is the Prinzian question of whether the philosopher who asks the question is doing this empirical work himself or herself or "works by citation".

4.1 First philosophy vs. second philosophy.

Whether empirical data should be taken into account for philosophical arguments is the crucial question for the debate not only between experimental philosophers and their critics, but also between naturalistically-minded philosophers in general (not all of whom subscribe to the paradigm of experimental philosophy[10]) and traditionally-minded philosophers. In the meta-discussion about experimental philosophy, the latter position is called *armchair philosophy*. The main argument of armchair philosophers against the use of empirical data is that it is not really dealing with the philosophical concepts themselves, but rather with usage of the terms corresponding to the concept in the general population:

> [Since philosophers generally assume] competence of the speaker, absence of performance errors, and basis in semantic rather than pragmatic considerations [...], intuition statements cannot be interpreted as straightforward predictions, and therefore cannot, for reasons of principle, be tested through the methods of non-participatory social science, without taking a stance on the concepts involved and engaging in dialogue. For example, when philosophers claim that according to our intuitions, Gettier cases are not knowledge, they are not presenting a hypothesis about gut reactions to counterfactual scenarios but, more narrowly, staking a claim of how competent and careful users of the ordinary concept of knowledge would pre-theoretically classify the case in suitable conditions. The claim, then, is not about what I will call surface intuitions but about robust intuitions, which are bound to remain out of reach of the Survey Model of experimentalists. (Kauppinen, 2007, p. 97)

Maddy (2007) introduces the term "second philosophy" for her position that she called "naturalism" in earlier work (Maddy, 1997), as she feels that the term 'naturalism' "has come to mark little more than a vague science-friendliness (p. 1)":[11]

> [The] Second Philosopher is equally at home in anthropology, astronomy, biology, botany, chemistry, linguistics, neuroscience, physics, physiology, psychology, sociology, ... and even mathematics, once she realizes how central it is to her ongoing effort to understand the world. [...] She simply begins from commonsense perception and proceeds from there to systematic observation, active experimentation, theory formation and testing, working all the while to assess, correct, and improve her methods as she goes. (Maddy, 2007, p. 2)

[10] Cf. (Papineau, 2015) for an in-depth discussion of different naturalistic positions in philosophy.

[11] "The Second Philosopher is a development of the naturalist in my (Maddy, 2001) and (Maddy, 2003) [...]; I adopt the name name here largely to avoid irrelevant debates about what 'naturalism' should be." (Maddy, 2007, p. 19, fn. 15)

As Maddy's terms suggest, the distinction is chiefly about whether philosophy or empirical observation have priority in the case of a conflict between the two. The first philosopher considers it possible that philosophical analysis results in a concept that, as an idealisation, may be in conflict with direct observations, and yet more worthy of philosophical study than the (pre-philosophical) everyday concept. In contrast, the second philosopher starts from the observation and would require a philosophical theory to explain it; a theory that does not match the observations would be scrutinised and possibly discarded:

> The theory of the real numbers, for example, is a fundamental component of the calculus and higher analysis, and as such is far more firmly supported than any philosophical theory of mathematical existence or knowledge. To sacrifice the former to preserve the latter is just bad methodology. (Maddy, 1990, p. 23)

We believe that the term *empirical philosophy* (or *empirically-based philosophy*) is best reserved for meta-philosophical positions that grant an important role for empirical data in the analysis of philosophical concepts and that would consider rejecting a philosophical theory if in clear conflict with empirical data. Note that this description does not specify the means by which these empirical data are collected. An empirical philosopher in this sense can espouse the experimental paradigm or reject it; he or she can consider qualitative methods of the social sciences or consider them too imprecise, etc.[12]

4.2 Empirical methodology.

Empirical data can be collected by several methods, ranging from unstructured observation via structured observation (using various methods) to experiments in controlled settings; some of the methods are qualitative, others quantitative.[13] The experimental method is so central for our contemporary idea of science that it requires constant reminders to recall that it is a relatively recent addition to the toolbox of the scientist and that it is only one among many empirical methods.[14] The relatively new field of "experimental philosophy" has mostly, but not exclusively, used the method of experiments. In the context of applying empirical findings to philosophy, a methodological discussion of the acceptable tools for data collection is very appropriate, especially since many of the philosophical applications involve empirical data from the social sciences where the choice of methodology is much more a matter of debate than in the exact sciences. Löwe and Van Kerkhove (in preparation) argue that a multiplicity of methods is particularly important in order to allow proper triangulation and that one should avoid relying solely on the experimental method. Obviously, it makes no sense to propose and change the name of the field *Experimental Philosophy* since it is well entrenched and accepted.[15]

[12] Note that in the above citation, Kauppinen (2007) only claims that "intuition statements [...] cannot [...] be tested through the methods of non-participatory social science", leaving it open whether empirical methods from the participatory social sciences might be able to serve as a test for intuition statements.

[13] We should like to emphasise that the experimental vs. non-experimental divide does not coincide with the quantitative vs. qualitative divide: most, but not all experimental work is quantitative and there are many quantitative non-experimental methods.

[14] The almost exclusive focus of philosophy of science on the experimental method has been criticized by some as an uncritical transfer of the method of modern physics to all of science (Lieberson and Lynn, 2002).

[15] It has to be conceded that this more liberal usage of the term "experimental" is not unique to the field of experimental philosophy: not all of "experimental physics" is strictly speaking experimental in methodology.

4.3 The source of empirical data.

The final distinction is the one highlighted by Prinz: he calls the philosopher who uses empirical data from the literature an "empirical philosopher" and the philosopher who does the empirical work herself or himself an "experimental philosopher". Using a sartorial analogue, what Prinz calls "empirical philosophy" is using empirical data like *ready-to-wear* or *off-the-rack* clothes, provided in the shelves and racks of a store with no direct input by the customer expressing his or her preferences of clothing style or fit; on the other end of the spectrum, one would have the *bespoke* experience where the customer can determine every detail of the garment and the garment is then tailored exactly to the specifications of the customer and made to fit his or her body perfectly. Those who have the appropriate skills could even become the tailors themselves, getting rid of any need to communicate wishes and desires. *Bespoke* empirical philosophy would be a project in which the philosopher works very closely with the empirical scientist and designs an experiment or other observational activity jointly with her or him; the extreme case of *bespoke* would be *do-it-yourself* where the philosopher becomes an empirical scientist and does the empirical work herself or himself. Of course, the more extravagant and non-standard your desires and wishes with respect to your clothes are, the less likely it is that you will find these off the rack and you might have to move towards bespoke tailoring. Similarly, the empirical philosopher cannot expect that sociologists, education researchers, cognitive scientists, and historians work on matters relating to questions of philosophical relevance without being explicitly prompted to do so.

When Weinberg et al. (2001) started wondering about the culture-independence of judgments in Gettier-like situations, they became *do-it-yourself* empirical philosophers by doing the experiments related on in their paper and interpreting the empirical results in the epistemological context. However, let us consider the following alternative history: suppose there was a group of cognitive psychologists or linguists who were independently interested in the question whether people from different cultural backgrounds use the word "knowledge" in Gettier-type situations differently and that this group produced the data from (Weinberg et al., 2001). Then Weinberg, Nichols and Stich could have published their paper as *ready-to-wear* empirical philosophy making the very same philosophical claim.

This *Gedankenexperiment* shows that whether the philosophical claim was made as *ready-to-wear* empirical philosophy or as *bespoke* empirical philosophy does not matter in principle for the quality of the philosophical argument. But, as in the sartorial word, not everything you need for your philosophical argument is available off the rack, forcing you to go bespoke or accept compromises in terms of fit between your data and the philosophical argument. The more you compromise on fit, the more it affects the quality of the philosophical argument; consequently, *ready-to-wear* empirical philosophy can only provide good arguments in a field where empirical data are sufficiently available to give the philosopher the resources to work with.

As in the case of tailoring, it is not universally the case that *bespoke* is better than *ready-to-wear*: if you find a high-quality garment off the rack that fits you very well, it may be considerable better than a bespoke garment from a mediocre tailor or (in the case of most of us with no expertise in tailoring) a garment that you made yourself. Similarly, in empirical philosophy, there are advantages and disadvantages to both *ready-to-wear* and *bespoke* approaches. In the first instance, philosophers are not empirical scientists, so *do-it-yourself* empirical philosophy requires that the philosopher acquire the skills and learn the techniques of another discipline, wasting time and energy that could be spent on something that they are more qualified for (such as doing philosophy) and possibly even leading to sub-standard or flawed empirical work. So, if the empirical data needed for a philosophical

argument exist in the literature and fit the argument well, then *ready-to-wear* empirical philosophy based on these data might be preferable to *bespoke* or *do-it-yourself* empirical philosophy.

An example of an area where adequate and appropriate data are available is philosophy of mind: a rich literature of empirical research is published by cognitive scientists, psychologists, and neuroscientists on human cognition and its interaction with the mind and the brain, allowing philosophers of mind to do *ready-to-wear* empirical philosophy of mind without starting collaborative projects with said cognitive scientists, psychologists and neuroscientists.[16]

5 The availability of data

In § 3, we have emphasized that it is chiefly the intended questions that distinguish *philosophy of mathematical practice* from the wider *study of mathematical cultures and practices*. In § 4, we introduced the (re-named) Prinzian distinction of *ready-to-wear* and *bespoke* empirical philosophy. In this terminology, *ready-to-wear* philosophers of mathematical practice would rely on published data of people who work on mathematical practices and cultures to do their philosophical arguments, whereas philosophers of mathematical practice going *bespoke* would be more genuinely involved in the wider community studying mathematical practices and cultures in the form of joint research projects with researchers in mathematics education, anthropology, sociology, history, cognitive science and other fields.

We emphasised that the main factor in deciding whether you go bespoke or not is the availability of data in the literature. In light of this, it is very relevant for us that empirical data on mathematical practices and cultures are scarce, and this has been lamented by members of our community for the last decade.[17]

One field where we have a reasonable amount of data on mathematical research practices is the history of mathematics. In cognitive science, there is a rich literature on number cognition,[18] but as soon as we move to higher cognitive aspects of mathematical reasoning, there is not much research available. Of course, in mathematics education, we have a large and thriving literature using empirical methods, but most of it focuses on primary and secondary school education and very few researchers in mathematics education deal with tertiary education, research education or research itself.[19] In the case of sociology, we have already speculated on the reasons of the lack of

[16] This is not to say that there is no *bespoke* empirical philosophy of mind; as examples, let us mention the collaboration of Newen and Vogeley (e.g., David et al., 2006, 2008; Kockler et al., 2010) or the collaboration of van Lambalgen with cognitive neuroscientists (e.g., Baggio et al., 2008; Pijnacker et al., 2011).

[17] The lament is already present in Kitcher and Aspray (1988, p. 17) in their introduction of the term "maverick tradition": "[B]ecause the [maverick] tradition is so recent, it now consists of a small number of scattered studies, studies that may not address the problems that are of most concern to mathematicians and historians." It can typically be found in the announcements of the events organised by our community; the following is a published version from (Löwe and Müller, 2010, p. vii): "[S]ociology of science mostly ignored mathematics presumably under the assumption that the human component of mathematical research is negligible."

[18] There is too much literature here to even give a few exemplary pointers; the two symposia *Mathematical Practice and Cognition* (organised by Alan Smaill, Markus Guhe, and Alison Pease) and *Mathematical Practice and Cognition II* (organised by Brendan Larvor and Alison Pease) at the 2010 and 2012 meetings of the Society for the Study of Artificial Intelligence and Simulation of Behaviour (AISB) in Leicester and Birmingham, respectively, got researchers in number cognition in touch with our community, and the special issue of the journal *Topics in Cognitive Science* (Volume 5, Issue 2, April 2013) containing the post-proceedings of the 2010 symposium shows the results of this cross-over nicely.

[19] Notable exceptions are, e.g., Weber and Mejia-Ramos (2011), Inglis and Alcock (2012), Inglis et al. (2013), and Weber et al. (2014).

literature on mathematics; Heintz (2000, p. 9) writes: "[d]ie Soziologie [begegnet] der Mathematik mit einer eigentümlichen Mischung aus Devotion und Desinteresse"; after Heintz's seminal book came out, a number of papers by members of our community have been published,[20] but compared to the sociology of other sciences, the literature is still very thin.

This forces empirical philosophers of mathematics to do one of two things: remain *ready-to-wear* empirical philosophers of mathematics and restrict their attention to those questions that can be discussed with the scarce data available, or become *bespoke* empirical philosophers of mathematics and start collaborative projects with the appropriate empirical scientists. Concerning the first option, historical empirical data on mathematical practices is much more available than empirical data from other neighbouring disciplines (such as sociology). We believe that this is one important factor in the perceived emphasis on historical studies in philosophy of mathematical practice.[21]

If a philosopher decides to do *bespoke* empirical philosophy of mathematics and to become actively involved in empirical research, it is important to notice that there is a large number of different disciplines involved in obtaining a multi-dimensional picture of mathematical practices and cultures. The number of involved disciplines is too large to hope that the philosopher could master all of the techniques from all of these disciplines; as a consequence, interdisciplinary collaboration with researchers from other fields is a necessary step for *bespoke* empirical philosophy of mathematics. This interdisciplinary collaboration in turn requires that the philosophers convince the researchers from other disciplines to get involved with their projects.

The following is an example of an effort to join forces with researchers from other disciplines by finding questions of relevance for everyone: In 2014 and 2015, the *International Union for History and Philosophy of Science and Technology* (IUHPST) ran a project *Cultures of Mathematical Research Training* funded by the *International Council of Science* (ICSU). This project brought researchers from all of the disciplines involved in our community together with representatives of funding agencies to produce a list of relevant research questions about the formation process of mathematical researchers that can be answered using empirical means. The project used a method for collaboratively identifying research priorities due to Sutherland et al. (2011). Since philosophy plays an important role in our community, philosophers were well represented during the two workshops of the project, guaranteeing that the philosophical legacy of our community does not get lost. Taking all of the represented disciplines into account, the project participants discussed which questions about the process of becoming a researcher in mathematics were the most relevant for the field as a whole. The resulting list of questions will be published as (Larvor and Löwe, 2016) and gives the result of this dialogue that required the philosophers to place their motivations into the larger context; it may serve as a catalyst for more interdisciplinary collaboration in the future.

6 Conclusion

We have argued that what we have called *our community* in § 2 can be seen as two things: the sub-community of the wider field of *the study of mathematical practices and cultures* of those people motivated by philosophical questions and at the same time the sub-community of philosophy of

[20] Cf., e.g., MacKenzie (2006), Greiffenhagen (2008), Rosental (2008), Greiffenhagen and Sharrock (2011a), Greiffenhagen and Sharrock (2011b), and Greiffenhagen (2014).

[21] It is difficult to measure emphasis on historical studies in a community; to give a rough indicator: at the first two APMP conferences (2010 in Brussels and 2013 in Urbana-Champaign), the percentage of abstracts explicitly mentioning the name of at least one pre-Second World War mathematician was 60% (17 out of 28) and 52% (13 out of 25), respectively.

mathematics that would be *empirical* in the sense of § 4. The term "philosophy of mathematical practice" emphasises the second characterisation. We have also discussed that the number of researchers actively working on mathematical practices and cultures that would be of relevance for philosophers of mathematics is small, and therefore, members of our community cannot expect to do *ready-to-wear* empirical philosophy of mathematics and "work by citation"; instead, they have to rely on close collaborations with cognitive scientists, researchers in mathematics education, sociologists, anthropologists, psychologists, and representatives of many other disciplines. In practice, we need to get these people excited about our questions and convince them that it is worthwhile to collaborate with us on questions. We might wonder whether the emphasis on questions driven by traditional philosophy of mathematics could constitute a practical obstacle in this endeavour.

References

Baggio, G., van Lambalgen, M., and Hagoort, P. (2008). Computing and recomputing discourse models: An erp study. *Journal of Memory and Language*, 59(1):36–53.

Buldt, B., Löwe, B., and Müller, T. (2008). Towards a new epistemology of mathematics. *Erkenntnis*, 68:309–329.

Corfield, D. (2003). *Towards a Philosophy of Real Mathematics*. Cambridge University Press.

David, A., Aumann, C., Santos, N., Bewernick, B., Eickhoff, S., Newen, A., and Vogeley, K. (2008). Differential involvement of the posterior temporal cortes in mental versus spartial perspective taking. *Social Cognitive and Affective Neuroscience*, 3:279–289.

David, N., Bewernick, B., Cohen, M., Newen, A., Lux, S., Fink, G., Shah, N., and Vogeley, K. (2006). The self-other distinction in social cognition—perspective-taking and agency in a virtual ball-tossing game. *Journal of Cognitive Neuroscience*, 18:898–910.

Dutilh Novaes, C. (2012). Towards a practice-based philosophy of logic: formal languages as a case study. *Philosophia Scientiae*, 16:71–102.

Ferreirós, J. (2014). Pragmatism in the philosophy of mathematics. unpublished talk given at the meeting *Pragmatism and the practical Turn in Philosophy of Science* of the *Académie Internationale de Philosophie des Sciences* held in Pont-à-Mousson, 10–14 September 2014.

Ferreirós, J. (2015). *Mathematical Knowledge and the Interplay of Practices*. Princeton University Press.

Greiffenhagen, C. (2008). Video analysis of mathematical practice? Different attempts to 'open up' mathematics for sociological investigation. *Forum: Qualitative Social Research*, 9(3):1–20.

Greiffenhagen, C. and Sharrock, W. (2011a). Does mathematics look certain in the front, but fallible in the back? *Social Studies of Science*, 41(6):839–866.

Greiffenhagen, C. and Sharrock, W. (2011b). Sources for myths about mathematics. on the significance of the difference between finished mathematics and mathematics-in-the-making. In François, K., Löwe, B., Müller, T., and Van Kerkhove, B., editors, *Foundations of the Formal Sciences VII. Bringing together Philosophy and Sociology of Science*, volume 32 of *Studies in Logic*, pages 91–109. College Publications.

Greiffenhagen, C. W. K. (2014). The materiality of mathematics: Presenting mathematics at the blackboard. *British Journal of Sociology*, 65(3):502–528.

Heintz, B. (2000). *Die Innenwelt der Mathematik. Zur Kultur und Praxis einer beweisenden Disziplin*. Springer.

Hersh, R. (1997). *What is mathematics, really?* Oxford University Press.

Inglis, M. and Alcock, L. (2012). Expert and novice approaches to reading mathematical proofs. *Journal for Research in Mathematics Education*, 43:358–390.

Inglis, M., Mejia-Ramos, J. P., Weber, K., and Alcock, L. (2013). On mathematicians' different standards when evaluating elementary proofs. *Topics in Cognitive Science*, 5(2):270–282.

Jullien, C. and Soler, L. (2014). Conceptions of mathematical practices: Some remarks. Commentary on "The impact of the philosophy of mathematical practice on the philosophy of mathematics", by Jean Paul Van Bendegem. In Soler, L., Zwart, S., Lynch, M., and Israel-Jost, V., editors, *Science After the Practice Turn in the Philosophy, History, and Social Studies of Science*, Routledge Studies in the Philosophy of Science, pages 227–237. Routledge.

Kauppinen, A. (2007). The rise and fall of experimental philosophy. *Philosophical Explorations*, 10(2):95–118.

Kitcher, P. and Aspray, W. (1988). An opinionated introduction. In Aspray, W. and Kitcher, P., editors, *History and Philosophy of Modern Mathematics*, volume XI of *Minnesota Studies in the Philosophy of Science*, pages 3–57. University of Minnesota Press.

Kockler, H., Scheef, L., Tepest, R., David, N., Bewernick, B. H., Newen, A., Schild, H. H., May, M., and Vogeley, K. (2010). Visuospatial perspective taking in a dynamic environment: Perceiving moving objects from a first-person-perspective induces a disposition to act. *Consciousness and Cognition*, 19(3):690–701.

Larvor, B. P. (2016). What are mathematical cultures? In Ju, S., Löwe, B., Müller, T., and Xie, Y., editors, *Cultures of Mathematics and Logic, Selected papers from the conference in Guangzhou, China, 9–12 November 2012*, Trends in the History of Science. Springer-Verlag. this volume.

Larvor, B. P. and Löwe, B. (2016). Cultures of mathematical research training. White Paper reporting on a project organised by the *International Union for History and Philosophy of Science and Technology*. in preparation.

Lieberson, S. and Lynn, F. B. (2002). Barking up the wrong branch: Scientific alternatives to the current model of sociological science. *Annual Review of Sociology*, 28:1–19.

Löwe, B. (2014). Mathematics and the new technologies. Part I: Philosophical relevance of a changing culture of mathematics. In Schroeder-Heister, P., Heinzmann, G., Hodges, W., and Bour, P. E., editors, *Logic, Methodology and Philosophy of Science, Proceedings of the 14th International Congress (Nancy), Logic and Science Facing the New Technologies*, pages 399–407. College Publications.

Löwe, B. and Müller, T., editors (2010). *PhiMSAMP. Philosophy of Mathematics: Sociological Aspects and Mathematical Practice*, volume 11 of *Texts in Philosophy*. College Publications.

Löwe, B., Müller, T., and Müller-Hill, E. (2010). Mathematical knowledge: a case study in empirical philosophy of mathematics. In Van Kerkhove, B., De Vuyst, J., and Van Bendegem, J. P., editors, *Philosophical Perspectives on Mathematical Practice*, volume 12 of *Texts in Philosophy*, pages 185–203. College Publications.

Löwe, B. and Van Kerkhove, B. (in preparation). Methodological triangulation in empirical philosophy of mathematics.

MacKenzie, D. (2006). Computers and the sociology of mathematical proof. In Hersh, R., editor, *18 Unconventional Essays on the Nature of Mathematics*, pages 128–146. Springer-Verlag.

Maddirala, N. (2014). Philosophy of logical practice: a case study in formal semantics. Master's thesis, Universiteit van Amsterdam. ILLC Publications MoL-2014-15.

Maddy, P. (1990). *Realism in Mathematics*. Oxford University Press.

Maddy, P. (1997). *Naturalism in Mathematics*. Oxford University Press.

Maddy, P. (2001). Naturalism: friends and foes. In Tomberlin, J., editor, *Philosophical Perspectives, Volume 15: Metaphysics*, pages 37–67. Blackwell.

Maddy, P. (2003). Second philosophy. *Journal of the Indian Council of Philosophical Research*, 20:73–106.

Maddy, P. (2007). *Second Philosophy. A naturalistic method*. Oxford University Press.

Mancosu, P. (2008a). Introduction. In Mancosu, P., editor, *The Philosophy of Mathematical Practice*, pages 1–21. Oxford University Press.

Mancosu, P., editor (2008b). *The Philosophy of Mathematical Practice*. Oxford University Press.

Marrou, H.-I. (1934). "*Doctrina*" et "*Disciplina*" dans la langue des pères de l'église. *Bulletin du Cange, Archivum Latinitatis Medii Aevi*, 9:5–25.

Müller-Hill, E. (2009). Formalizability and knowledge ascriptions in mathematical practice. *Philosophia Scientiae*, 13(2):21–43.

Müller-Hill, E. (2011). *Die epistemische Rolle formalisierbarer mathematischer Beweise—Formalisierbarkeitsorientierte Konzeptionen mathematischen Wissens und mathematischer Rechtfertigung innerhalb einer sozio-empirisch informierten Erkenntnistheorie der Mathematik*. PhD thesis, Rheinische Friedrich-Wilhelms-Universität Bonn.

Papineau, D. (2015). Naturalism. In Zalta, E. N., editor, *The Stanford Encyclopedia of Philosophy*. Fall 2015 Edition.

Pijnacker, J., Geurts, B., van Lambalgen, M., Buitelaar, J., and Hagoort, P. (2011). Reasoning with exceptions: an event-related brain potentials study. *Journal of Cognitive Neuroscience*, 23(2):471–480.

Prinz, J. J. (2008). Empirical philosophy and experimental philosophy. In Knobe, J. and Nichols, S., editors, *Experimental Philosophy*, pages 198–208. Oxford University Press.

Rosental, C. (2008). *Weaving Self-Evidence. A Sociology of Logic*. Princeton University Press.

Stokhof, M. and van Lambalgen, M. (2011). Abstractions and idealisations: The construction of modern linguistics. *Theoretical Linguistics*, 37(1-2):1–26.

Sutherland, W. J., Fleishman, E., Mascia, M. B., Pretty, J., and Rudd, M. A. (2011). Methods for collaboratively identifying research priorities and emerging issues in science and policy. *Methods in Ecology and Evolution*, 2(3):238–247.

Van Bendegem, J. P. (2014). The impact of the philosophy of mathematical practice on the philosophy of mathematics. In Soler, L., Zwart, S., Lynch, M., and Israel-Jost, V., editors, *Science After the Practice Turn in the Philosophy, History, and Social Studies of Science*, Routledge Studies in the Philosophy of Science, pages 215–226. Routledge.

Van Kerkhove, B., editor (2008). *New Perspectives On Mathematical Practices. Essays in Philosophy and History of Mathematics. Brussels, Belgium, 26–28 March 2007*. World Scientific.

Van Kerkhove, B., De Vuyst, J., and Van Bendegem, J. P., editors (2010). *Philosophical Perspectives on Mathematical Practice*, volume 12 of *Texts in Philosophy*. College Publications.

Van Kerkhove, B. and Van Bendegem, J. P., editors (2007). *Perspectives on Mathematical Practices. Bringing Together Philosophy of Mathematics, Sociology of Mathematics, and Mathematics Education*, volume 5 of *Logic, Epistemology, and the Unity of Science*. Springer-Verlag.

Weber, K., Inglis, M., and Mejia-Ramos, J. P. (2014). How mathematicians obtain conviction: Implications for mathematics instruction and research on epistemic cognition. *Educational Psychologist*, 49:36–58.

Weber, K. and Mejia-Ramos, J. P. (2011). Why and how mathematicians read proofs: an exploratory study. *Educational Studies in Mathematics*, 76(3):329–344.

Weinberg, J. M., Nichols, S., and Stich, S. (2001). Normativity and epistemic intuitions. *Philosophical Topics*, 29:429–460.